Unit 59: Advanced Mathematics for Engineering

Unit code: K/601/1412

QCF level: 5

Credit value: 15

Aim

This unit aims to provide the analytical knowledge necessary for studying engineering to degree level and will provide the more advanced knowledge required for a range of careers in engineering.

Unit abstract

This unit will enable learners to develop further techniques for the modelling and solution of engineering problems.

Learners will review methods for standard power series and use them to solve ordinary differential equations. Numerical methods are then considered before both methods are used to model engineering situations and determine solutions to those equations.

Laplace transforms are introduced in learning outcome 2 and their use in solving first and second order differential equations together with the solution of simultaneous equations.

In learning outcome 3, Fourier coefficients are determined to represent periodic functions as infinite series and then the Fourier series approach is applied to the exponential form to model phasor behaviour. The final part of this learning outcome involves using the Fourier series to model engineering situations and solve problems.

Learning outcome 4 reviews partial differentiation techniques to solve rates of change problems and problems involving stationary values. Also in this learning outcome, direct partial integration and the separation of variables methods are used to solve partial differential equations. Finally, partial differential equations are used to model engineering situations and solve problems.

Learning outcomes

On successful completion of this unit a learner will:

1 Be able to analyse and model engineering situations and solve engineering problems using series and numerical methods for the solution of ordinary differential equations

2 Be able to analyse and model engineering situations and solve engineering problems using Laplace transforms

3 Be able to analyse and model engineering situations and solve engineering problems using Fourier series

4 Be able to analyse and model engineering situations and solve engineering problems using partial differential equations.

Unit content

1 **Be able to analyse and model engineering situations and solve engineering problems using series and numerical methods for the solution of ordinary differential equations**

Power series: review of methods for standard series, Maclaurin's series and Taylor's series

Power series methods: methods eg higher differential coefficients and Leibnitz's theorem, recurrence relations, Leibnitz–Maclaurin method, Frobenius method, engineering use of Bessel's equation and Legendre equation, Bessel functions of the first and second kind, Legendre's equation and polynomials

Numerical methods: restrictions on the analytical solution of differential equations; typical methods eg Taylor's series, solution of first order differential equations, Euler's method, improved Euler method, Runge–Kutta method

Engineering situations: model engineering situations and solve problems using ordinary differential equations eg vibration, thermofluids and heat transfer, mechanics of solids, electrical systems, information systems

2 **Be able to analyse and model engineering situations and solve engineering problems using Laplace transforms**

Laplace transform: use of Laplace transform; transforms of standard functions; first shift theorem; inverse transforms and tables of inverse transforms; transforms using partial fractions; poles and zeros; solution of first and second order differential equations using Laplace transforms; solution of simultaneous differential equations; initial and final value problems

Engineering situations: model engineering situations and solve problems using Laplace transforms eg electrical circuits in the *s*-domain, modelling and analysis of closed loop control systems, response of first and second order systems, servomechanisms, systems engineering, systems stability analysis, automatic flight control systems, design of feedback systems – root locus plots, Nyquist and Bode plots, Nichols charts

3 **Be able to analyse and model engineering situations and solve engineering problems using Fourier series**

The Fourier series: sinusoidal and non-sinusoidal waveforms; periodic functions; harmonics; the Fourier series; Fourier coefficients; series for common wave-forms; odd and even functions and their products; half-range series; non-periodic functions and their half-range series

The exponential form: complex notation; symmetry relationship; frequency spectrum and phasors

Engineering situations: model engineering situations and solve problems using Fourier series eg electric circuit analysis, root mean square values, power and power factors, numerical integration and numerical harmonic analysis

4 **Be able to analyse and model engineering situations and solve engineering problems using partial differential equations**

Partial differentiation: review of partial differentiation techniques; partial differentiation and rates of change problems; change of variables; stationary values and saddle points

Partial differential equations: definition of partial differential equations; partial integration; solution by direct partial integration; initial conditions and boundary conditions; solution by separation of variables

Engineering situations: model engineering situations and solve problems using partial differential equations eg the wave equation and its application to vibration, the heat conduction equation, the Laplace equation and its application to temperature and potential

Learning outcomes and assessment criteria

Learning outcomes **On successful completion of this unit a learner will:**	**Assessment criteria for pass** **The learner can**:
LO1 Be able to analyse and model engineering situations and solve engineering problems using series and numerical methods for the solution of ordinary differential equations	1.1 determine power series values for common scientific and engineering functions 1.2 solve ordinary differential equations using power series methods 1.3 solve ordinary differential equations using numerical methods 1.4 model engineering situations, formulate differential equations and determine solutions to these equations using power series and numerical methods
LO2 Be able to analyse and model engineering situations and solve engineering problems using Laplace transforms	2.1 determine Laplace transforms and their inverse using tables and partial fractions 2.2 solve first and second order differential equations using Laplace transforms 2.3 model and analyse engineering systems and determine system behaviour using Laplace transforms
LO3 Be able to analyse and model engineering situations and solve engineering problems using Fourier series	3.1 determine Fourier coefficients and represent periodic functions as infinite series 3.2 apply the Fourier series approach to the exponential form and model phasor behaviour 3.3 apply Fourier series to the analysis of engineering problems 3.4 use numerical integration methods to determine Fourier coefficients from tabulated data and solve engineering problems using numerical harmonic analysis
LO4 Be able to analyse and model engineering situations and solve engineering problems using partial differential equations	4.1 solve rates of change problems and problems involving stationary values using partial differentiation 4.2 solve partial differential equations using direct partial integration and separation of variables methods 4.3 model and analyse engineering situations using partial differential equations.

Guidance

Links

This unit is intended to link with and extend the knowledge gained from studying *Unit 35: Further Analytical Methods for Engineers*.

Essential requirements

There are no essential requirements for this unit.

Employer engagement and vocational contexts

Delivery of this unit will benefit from centres establishing strong links with employers willing to contribute to the delivery of teaching, work-based placements and/or detailed case study materials.